Theory of turbocompressors

A course of lectures. 3 semester.
Group 63238/10

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Theory of turbocompressors. *Lecture 5.*

CONTENT

Thermodynamic and gas dynamic basics of theory of turbocompressors

- **Energy conservation equation in "heat" form (energy equation in gas flow).**
- **Continuity equation.**
- **Polytrophic coefficient connection with process parameters.**
- **Adiabatic, polytrophic and isothermal static and total parameters efficiency.**
- **Flow parameters changing characters in stage flow passage – basic equations usage with static and total parameters accounting.**

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Energy equation

Gas flow energy equation in "**heat**" form is another form of energy conservation and also is adduced in specific units, or which belong to gas unit-mass:

$$i_1 + 0{,}5c_1^2 + h_i + q_{ex} = i_2 + 0{,}5c_2^2$$

Energy equation

The sum of colorific content (enthalpy) $i=c_pT$ and kinetic energy $0,5c^2$ when gas is moving, could be changed only when the external mechanical work h_i (internal "head") and external heat q_{ex} is inputted or outputted

Energy equation

$i + 0{,}5c^2 = i^*$ - **stagnation enthalpy** determine decisively the unit-mass of gas energy. Using this value we can imagine energy equation as:

$$h_i + q_{ex} = i_2^* - i_1^* = c_p(T_2^* - T_1^*),$$

h_i - **internal "head"** – mechanical work which is passed to the gas unit-mass when it is moving through the impeller;

$T^* = i^*/c_p = T + 0{,}5c^2/c_p$ **- stagnation temperature**

Energy equation

Continuity equation – is one of the forms of the law of conservation of matter

$$\overline{m} = \rho \overline{V} = \rho f c = \text{const}$$

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Continuity equation

Gas mass flow through the control section at flow passage is the same, if between the control sections there is no additional input of output of gas mass flow.

Gas density at the impeller and diffusers of turbocompressors is raising, that is why the volume consumption $\bar{V} = \bar{m} / \rho$ becomes less with the gas moving through the flow passage.

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Polytrophic and adiabatic efficiency by static and total parameters

For the flow passage gasodynamic perfection characterization without the change of kinetic energy accounting **polytrophic and adiabatic efficiency by static and total parameters** is being used.

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Polytrophic and adiabatic efficiency by static and total parameters

Polytrophic efficiency is determined as ratio of effective polytrophic work of compression to the internal "head" which has been passed to the compressed gas without dynamic "head".

$$\eta_p = \frac{h_p}{h_i - h_d} = \frac{h_p}{h_p + h_r}$$

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Polytrophic and adiabatic efficiency by static and total parameters

Polytrophic efficiency is joint with polythropic exponent by the ratio:

$$\frac{n}{n-1} = \eta_p \frac{k}{k-1}$$

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Polytrophic and adiabatic efficiency by static and total parameters

In adiabatic efficiency in in distinction from the polytrophic there is no additional work in useful work because of the gas heating up in consequence of heat q_r input

$$\eta_{ad} = \frac{h_{ad}}{h_i - h_{id}} = \frac{h_{ad}}{h_p + h_r}$$

As for the compressor had<hp adiabatic efficiency of the compressor is always less then polytrophic efficiency: $\eta_{ad} < \eta_p$

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Polytrophic and adiabatic efficiency by static and total parameters

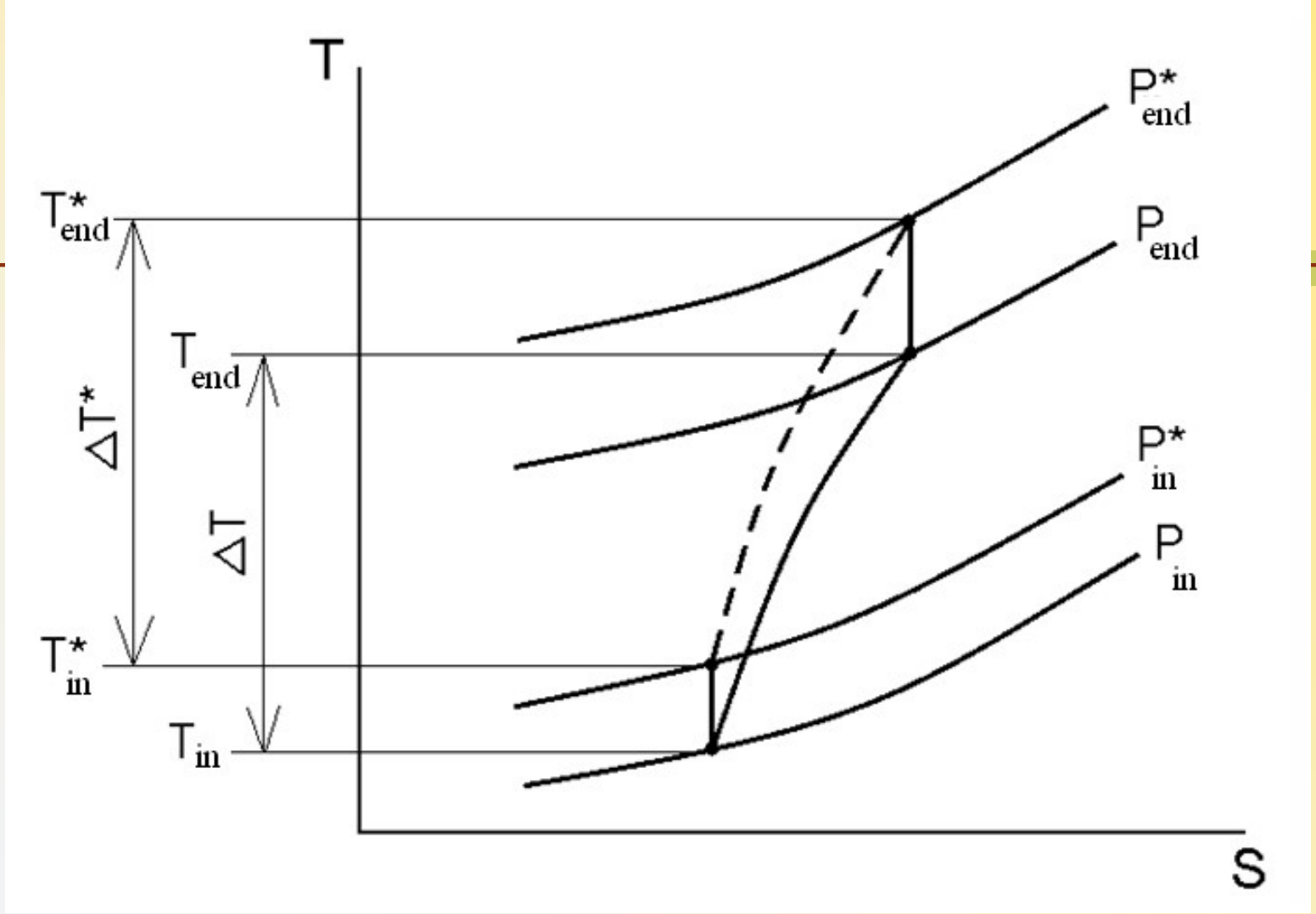
The drawback of the efficiency by static parameters is that it doesn't take into account the kinetic energy change and couldn't be used for efficiency of the flow passage estimation in case if dynamic "head" is essentially changing and it couldn't be considered as negligible in accordance with polytrophic "head". For the gasodynamic efficiency consideration the efficiency by total parameters is used, ratio of the sum of useful polytrophic compression work and dynamic "head" to internal "head".

Polytrophic and adiabatic efficiency by static and total parameters

Polytrophic efficiency by total parameters:

$$\eta_p^* = \frac{h_p + h_d}{h_i} = \frac{h_p^*}{h_p^* + h_r}$$

where $h_p^* = h_p + h_d$ - polytrophic "head" by total parameters



Real polytrophic compression process (continuous line) and compression process by total parameters (dashed line)

$$c_p \Delta T^* = h_i \qquad c_p \Delta T = h_i - h_{in}$$

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Isothermal efficiency of compressor. Gas cooling and compression

Isothermal compression process is considered as ultimate ideal process for turbocompressors with gas cooling while compressing. For this the heat which is equivalent to input mechanical work should be outlet by means of external heat exchange (energy equation at gas flow):

$$q_{\text{in}} = -h_i,$$ then $$T_2 = T_1$$

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Isothermal efficiency of compressor. Gas cooling and compression

For energy efficiency valuation isothermal efficiency is used, which considers not only gasodynamic but also cooling efficiency.

$$\eta_{is} = \frac{h_{is}}{h_i} = \frac{RT_1 \ln(\pi)}{c_p(T_2^* - T_1^*)}$$

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Isothermal efficiency of compressor. Gas cooling and compression

Isothermal efficiency always less then polytrophic efficiency of compressor, because it accounts not only gasodynamic losses but also the level of nonisothermality (in contrast to ideal, in real cooled compressor temperature is increasing):

$$\eta_{is} < \eta_{ad} < \eta_{p}$$

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The polytrophic exponent and process parameters connecting

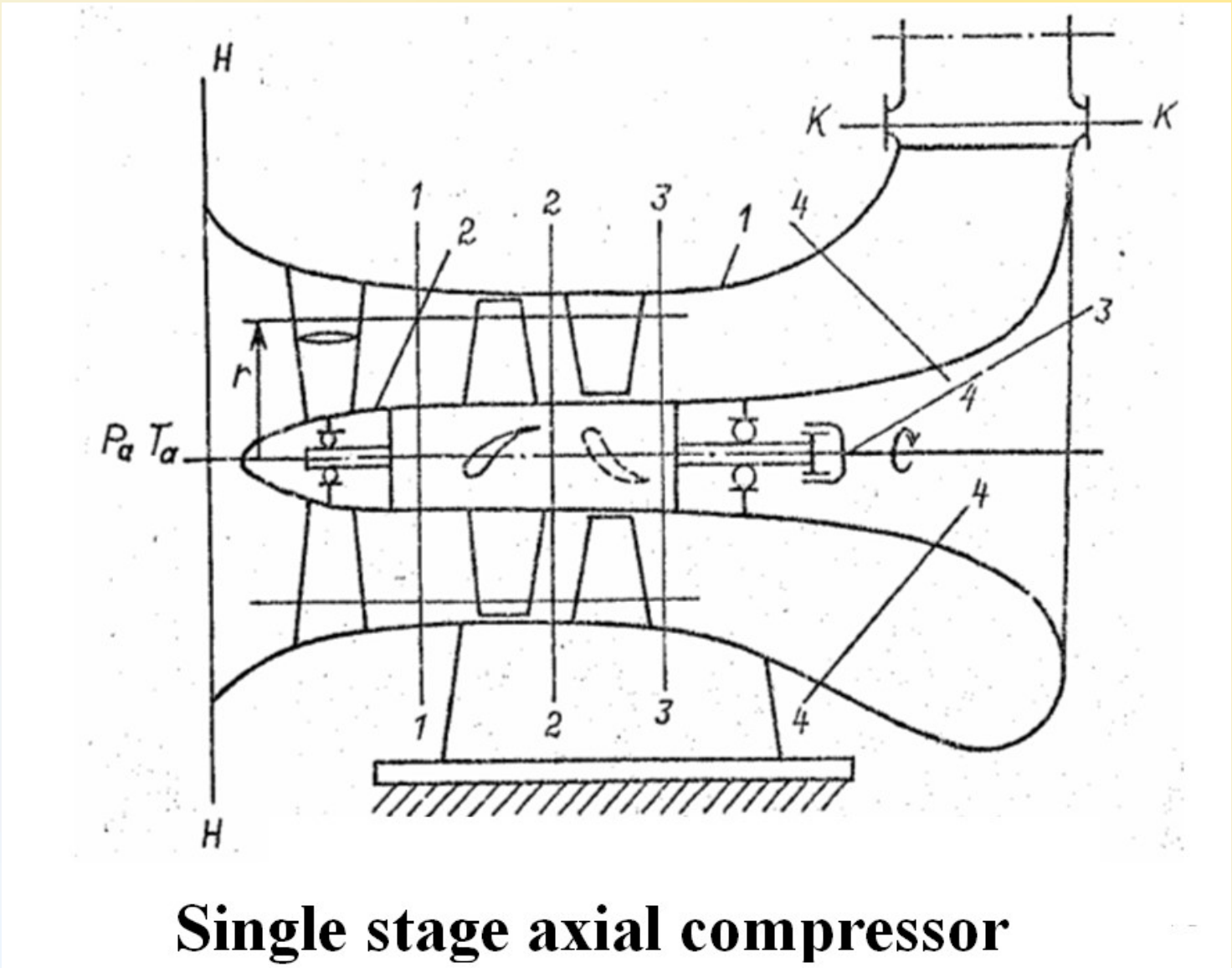
The connection of polytrophic exponent with process parameters is determined through the ratio:

$$\frac{n}{n-1} = \eta_p \frac{k}{k-1}$$

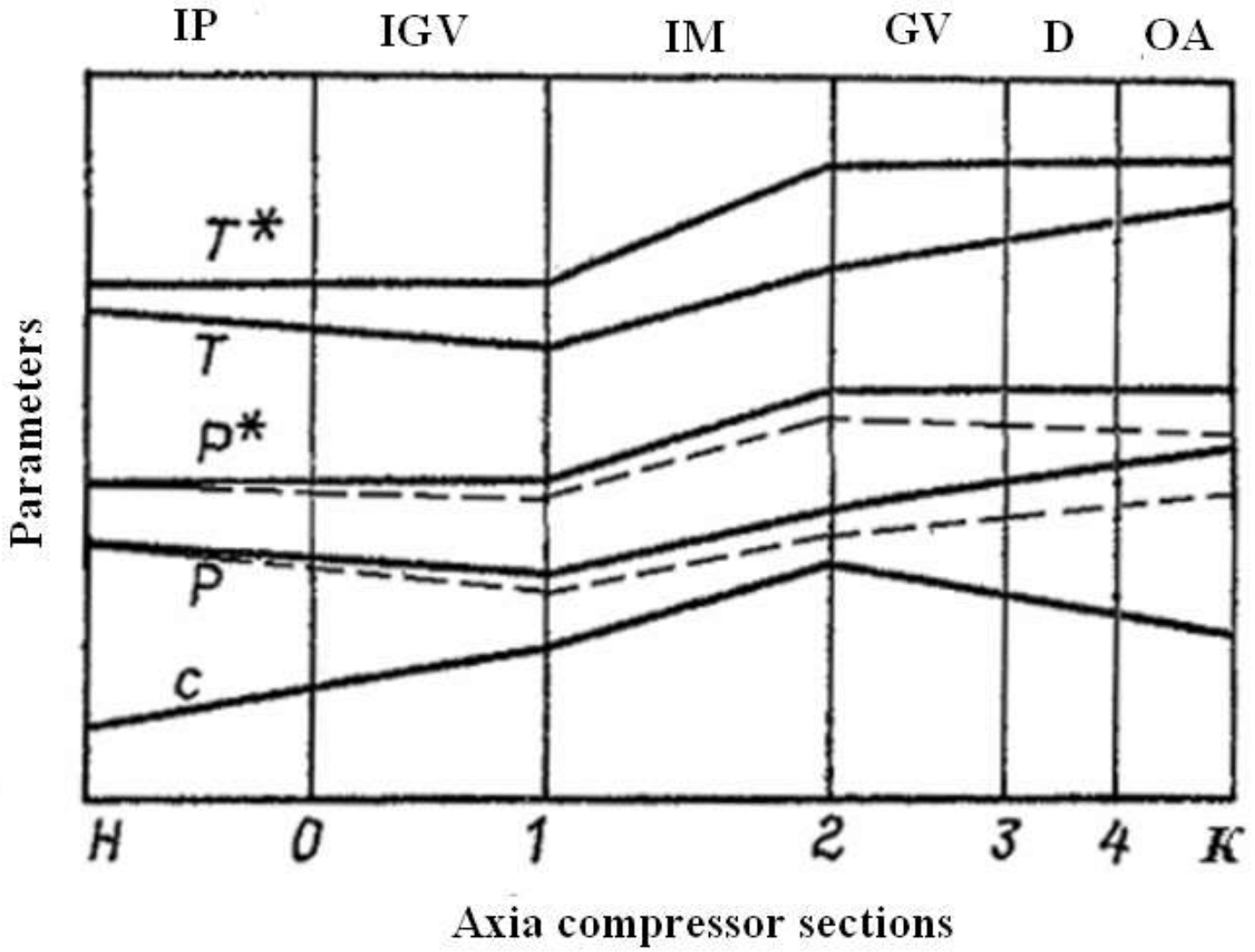
Process parameters ***(c, p, T)*** are included to polytrophic efficiency η_p

The connection of polytrophic exponent with process parameters is shown on ***i-S*** and ***T-S*** diagrams

Changing of gas parameters in the flow passage of ideal and real compressor



Single stage axial compressor



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Flow parameters changing in the flow passage of AC by sections: continuous line – ideal compressor; dashed – real compressor. $h_r=0$, $q_{ex} = 0$

Changing of gas parameters in the flow passage of ideal and real compressor

General for all turbomashines conclusion:
(and for all others flow case)

1. At all the canals where there is no mechanical energy input, total pressure decreases during gas moving in canal. Such canals are all turbomachines canals (confusers and diffusers), except impellers canals between the blades of turbocompressors.

2. Total pressure at the impellers increases because of internal "head" hi, but to the value which is less then at ideal compressor.

3. Because of less value p* static pressure in real TC is also less then in relevant ideal TC.

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CONTENT

Turbocompressor's stage basic theory

- Stage characteristics (compressor) as a combination of possible working modes when different network state.
- Characteristic's form in example of centrifugal stage.
- "Head" characteristics of the stage of ideal compressor with impeller which has infinite number of blades.
- Theoretical and internal "heads". "Head" coefficient. Mass flow coefficient.
- Dimensionless and dimension parameters connection.

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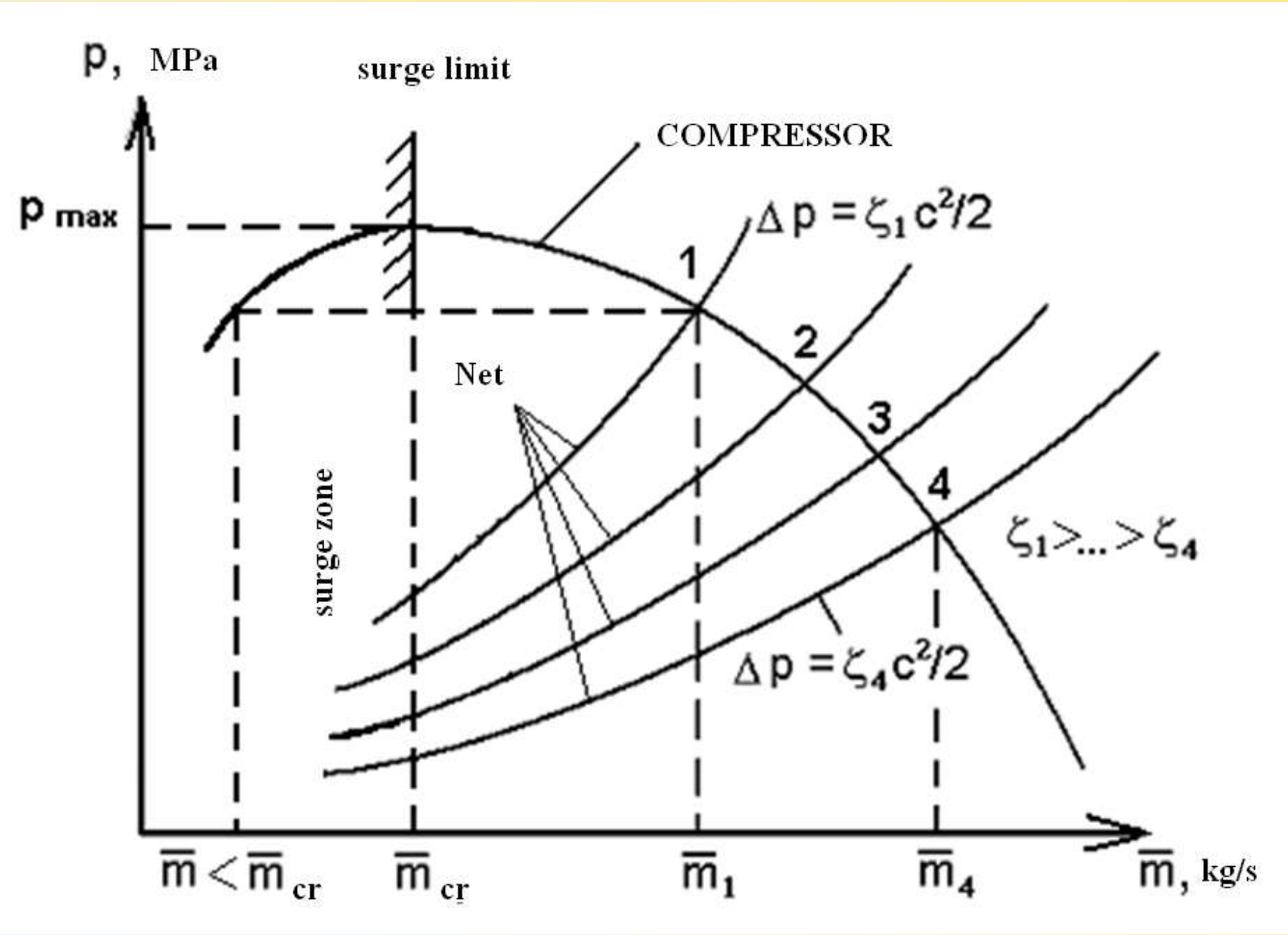
Turbocompressors characteristics as a assembly of practicable working modes when different network states.

Compressor's or stage characteristic determines modes when their work is practicable. Knowing the characteristic of network we can estimate on which particular mode the compressor will work (network is a compressed gas consumer).

Term of combined work with network: when the mass flow is the same with network compressor should reach the pressure, which is needed for mass flow moving through the network.



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Turbocompressor's "head" characteristics with infinite number of blades.

"Head" characteristics is a "head" dependence from the productivity.

While analizing compressor characteristics it is worth to use dimensionless ratios, what is a powerful source of the analyze results generalization.

Dimensionless geometric parameters are determined by D2 division (external diameter of the impeller).

Dimensionless velocities are determined by u2 derivation (circular velocity of the impeller)

Dimensionless velocities are marked by upper line:

$$\overline{c}_{u2} = c_{u2} / u_2$$

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Turbocompressor's "head" characteristics with infinite number of blades. "Head" coefficient.

Theoretical "head" coefficient of the impeller is a ratio of theoretical "head" to circular outlet from the impeller velocity:

$$\psi_t = h_t / u_2^2$$

As $h_t = c_{u2}u_2 - c_{u1}u_1,$

When there is no swirl of the flow on the inlet of impeller ($c_{u1}=0$):

$$\psi_t = h_t / u_2^2 = c_{u2}u_2 / u_2^2 = \overline{c}_{u2}$$

Turbocompressor's "head" characteristics with infinite number of blades. "Head" coefficient.

Internal "head" coefficient:

$$\psi_i = h_i / u_2^2 = \psi_T (1 + \beta_{leak} + \beta_{frict})$$

Polytrophic (useful) "head" coefficient:

$$\psi_p = h_p / u_2^2$$

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Internal capacity:

$$N_i = N_t + N_{fr} + N_{leak}$$

N_{fr} – Disks friction capacity

N_{leak} – Gas leakage capacity

Internal capacity, refered to N_t:

$$N_i/N_t = 1 + \beta_{leak} + \beta_{fr},$$

where $\beta_{fr} = N_{fr}/N_t;$

$$\beta_{leak} = \bar{m}_{leak} h_t / \bar{m} h_t = \bar{m}_{leak} / \bar{m}$$

-dimentionless coefficient of disk friction and leakage through the sealing

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Turbocompressor's "head" characteristics with infinite number of blades. "Head" coefficient.

In the Bernoulli's equation the internal "head" h_i could be imagined as sum of theoretical (Euler's) "head" h_t, "head" of disk friction h_{fr} and "head" of leakage h_{leak}.

$$h_i = h_t + h_{fr} + h_{leak}$$

$$h_i/h_t = 1 + \beta_{fr} + \beta_{leak}$$

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Turbocompressor's "head" characteristics with infinite number of blades. "Head" coefficient.

At the Bernoulli's equation lost "head":

$$h_r = h_w + h_{fr} + h_{leak}$$

where h_w – hydraulic losses

Bernoulli's equation could be imagined as processes directly in the flow passage:

$$h_t = h_p + h_d + h_w$$

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Turbocompressor's "head" characteristics with infinite number of blades. Consumption coefficient.

Consumption coefficient is ratio of consumption part of velocity to characteristic circular velocity u2:

$$\varphi = c_m / u_2 = \overline{c}_m$$

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For centrifugal compressors stages besides the consumption coefficient φ special consumption coefficient Φ is widely used:

$$\Phi = \frac{\bar{m}}{\rho_{in}^{*}\left(\pi D_2^2 / 4\right) u_2}$$

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The dependence of special consumption coefficient Φ could be determined through the consumption coefficient φ2:

-mass flow is determined through the volume consumption V2 and gas density ρ_2^* at the outlet of the impeller by ratio $\bar{m} = \rho_2^* \bar{V}_2$;

-- volume consumption V2 could be determined through the consumption part of gas velocity c_{r2} at the outlet of impeller and outlet square of impeller outlet F_2

-- outlet square of impeller is determined by $F_2 = \pi D_2 b_2$

$$\Phi = \frac{\bar{m}}{\rho_н^* \left(\frac{\pi D_2^2}{4}\right) u_2} = \frac{\rho_2^* \bar{V}_2}{\rho_н^* \left(\frac{\pi D_2^2}{4}\right) u_2} = \frac{\rho_2^* c_{r2} F_2}{\rho_н^* \left(\frac{\pi D_2^2}{4}\right) u_2} =$$

$$= \frac{\rho_2^* c_{r2} \pi D_2 b_2}{\rho_н^* \left(\frac{\pi D_2^2}{4}\right) u_2} = \frac{4 \rho_2^* c_{r2} b_2}{\rho_н^* u_2 D_2} = 4 \varepsilon_2 \varphi_2 \bar{b}_2$$

$$\Phi = 4 \varepsilon_2 \varphi_2 \bar{b}_2$$

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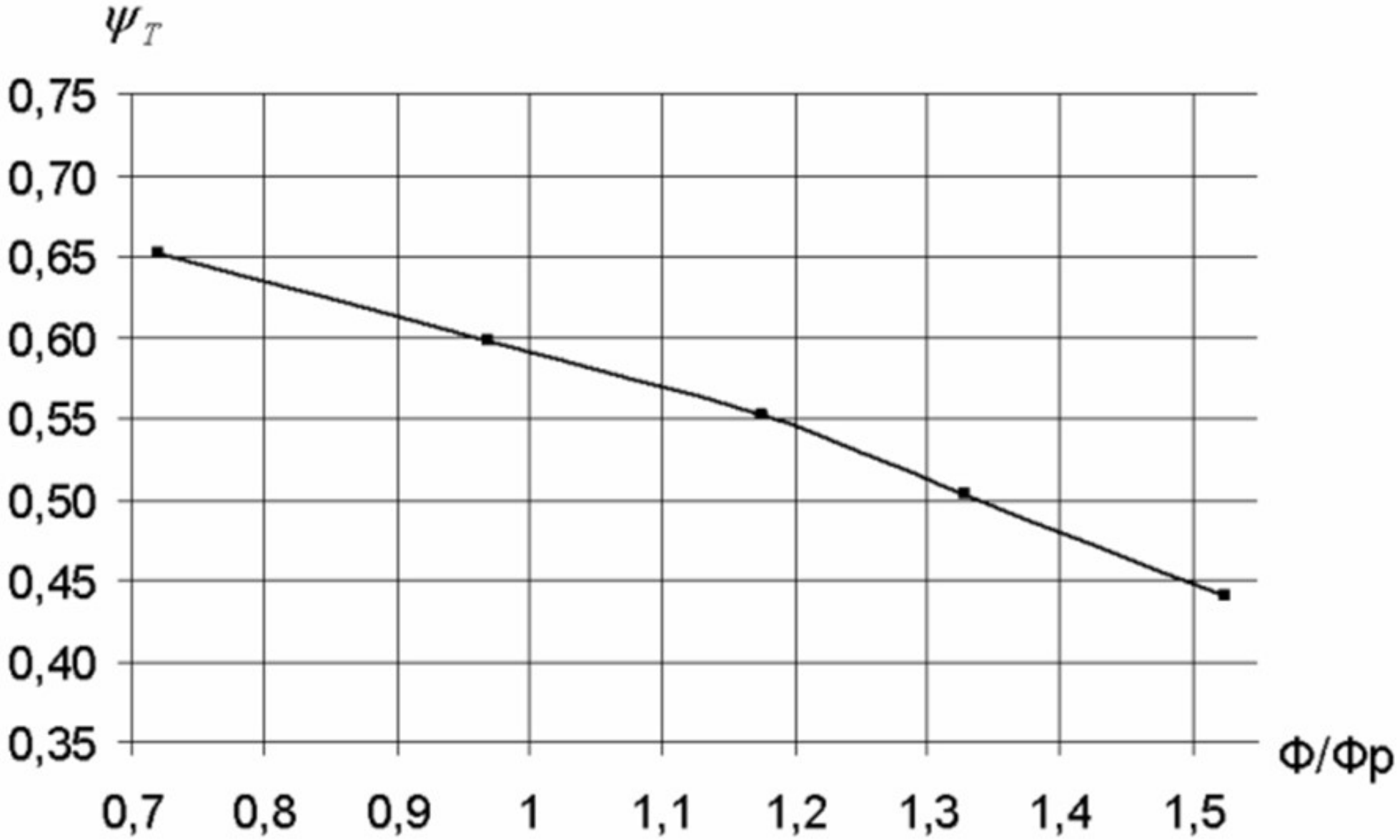

Stage characteristic of CC

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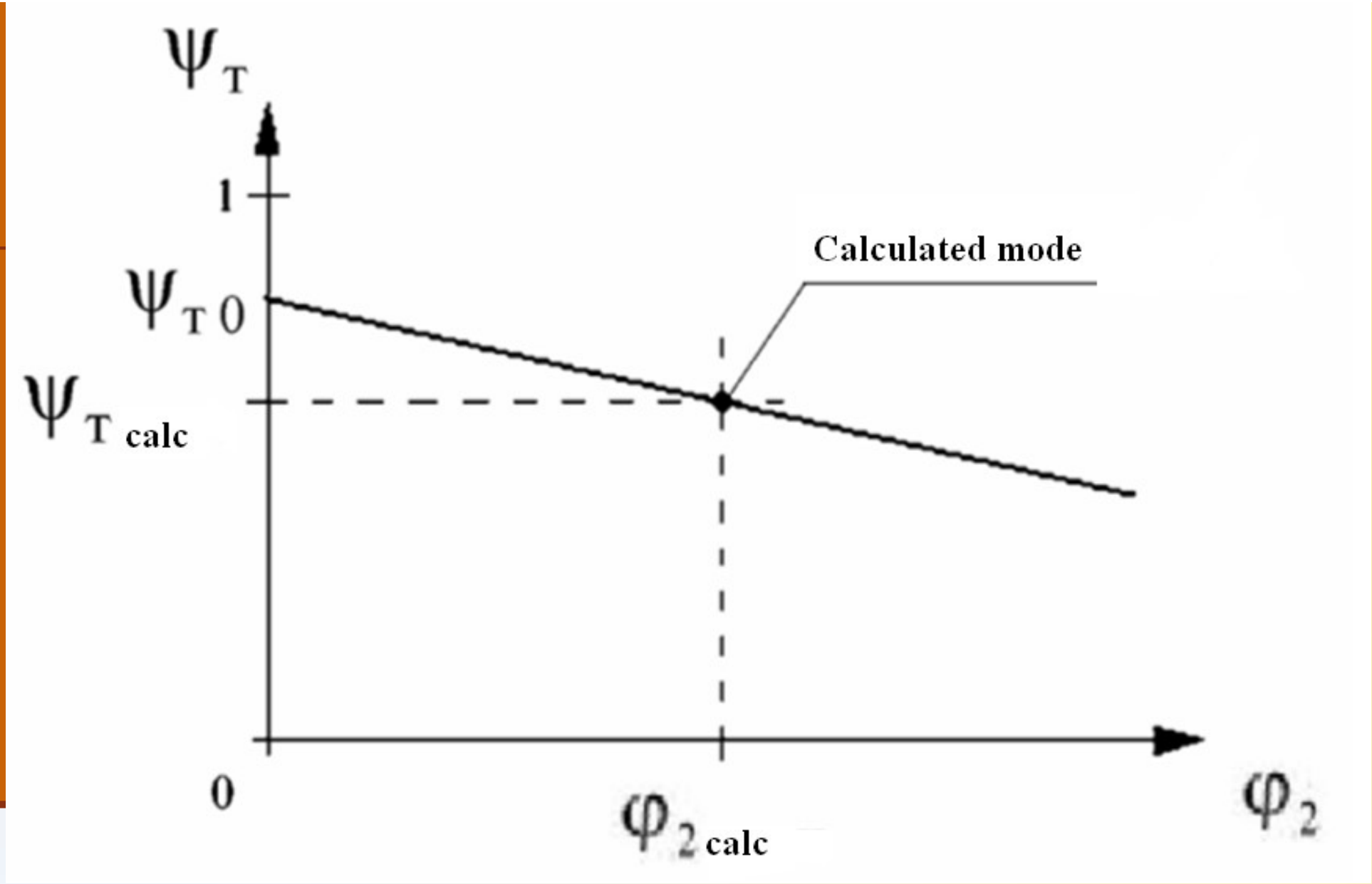

Linear dependence of the theoretical "head" coefficient from the impeller consumption graph

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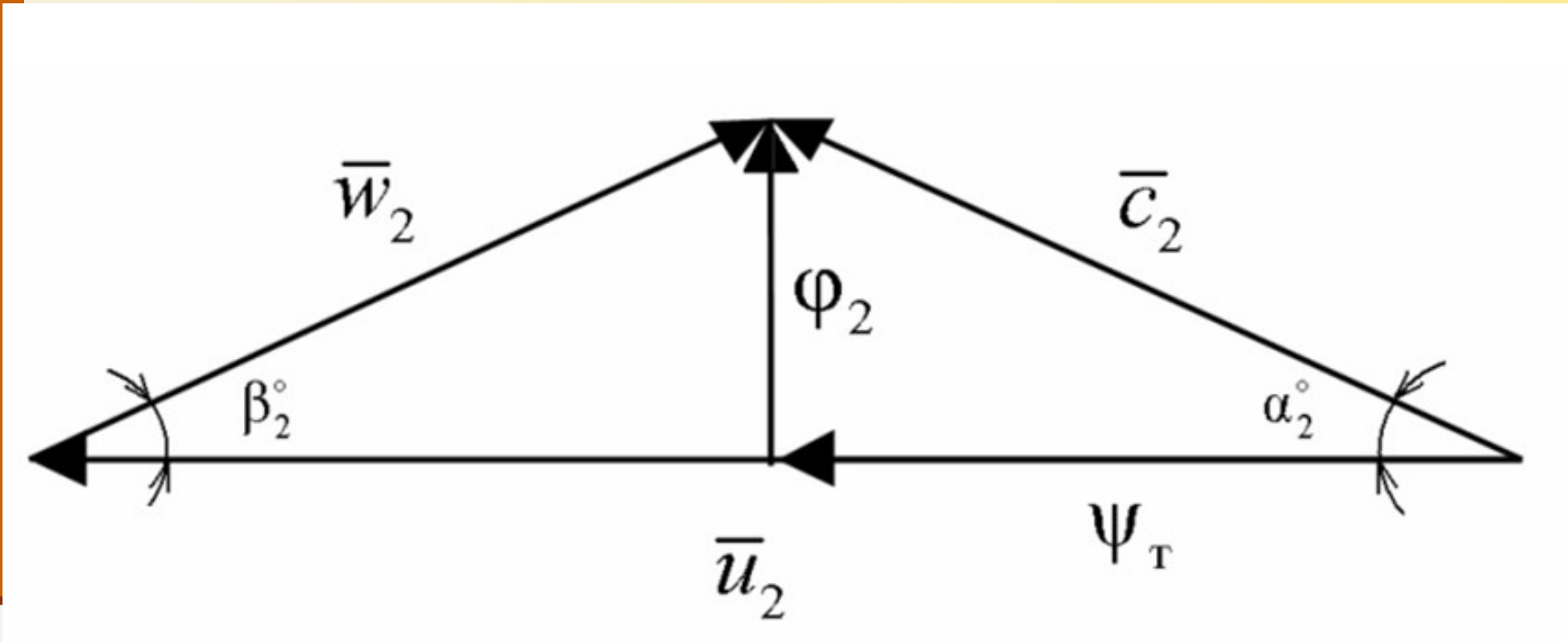
Turbocompressor's "head" characteristics with infinite number of blades.



Dimensionless velocity triangle

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Dimensionless and dimension parameters connection

$$\bar{m} = \rho_2 \bar{V}_2 = \frac{p_2}{RT_2} 2\pi r_2 b_2 u_2 \varphi_2$$

and

$$p_к = p_н \pi = p_н \left[(k-1) \psi_т \frac{u_2^2}{kRT_н} + 1 \right]^{\eta_п \frac{k}{k-1}}$$

as

$$h_т = \psi_т u_2^2 = \frac{k}{k-1} RT_н \left[\pi^{\frac{k-1}{\eta_п k}} - 1 \right]$$

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In gasodynamic the compression criteria is widely spread – **Mach number**:

$$M = c/a$$

Where ***c*** – gas velocity, ***a*** – sound velocity.
There is a straight analogy between physic number and **special Mach number**:

$$M_u = u_2 / \sqrt{kRT_{in}}$$

Which is widely spread in TC theory

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Mach number

In the Mach number the u2 velocity determines used specific work on gas consumption, and sound velocity at the inlet section $a_{in} = \sqrt{kRT_{in}}$ - gas elasticity. **Pressure ratio is determined by dimensionless values:**

-"head" coefficient,
-efficiency,
-Mu number,
-adiabatic exponent k:

$$\pi = \frac{p_{out}}{p_{in}} = \left[1 + (k-1)\psi_T M_u^2\right]^{\frac{k}{k-1}\eta_p}$$

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